COMPARISON OF THE DIRECTIONAL POINT SOURCE MODEL AND BEM MODEL FOR ARRAYED LOUDSPEAKERS

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1 INTRODUCTION

There are many approximations to the Acoustic wave equation which can be evaluated numerically which are more and less accurate at modeling different acoustical phenomena and which are more or less computationally expensive. This paper compares the Directional Point Source model and the Boundary Element Method (BEM) for modeling arrayed loudspeakers.

1.1 Directional Point Source Model

Given the linear wave equation for pressure:

$$\nabla^2 p = \frac{1}{c^2}\frac{\partial^2 p}{\partial t^2}$$

where p is the acoustic pressure, t is time, and c is the sound speed, one can derive that the time-harmonic pressure radiated by a sphere vibrating radially is in the form:

$$p(r,t) = \frac{A}{r}e^{j(\omega t - kr)}$$

where r is the distance from the center of the sphere, A is a complex magnitude, ω is the angular frequency, and k is the spatial wave number[1]. This is the exact expression for radiation from a point source. Since the wave equation is linear, the pressure radiated from a combination of point sources is simply the linear superposition of the pressure radiated by the individual point sources. For example the exact pressure radiated by two point sources is:

$$p_d(r,t) = \frac{A_1}{r_1}e^{j(\omega t - kr_1)} + \frac{A_2}{r_2}e^{j(\omega t - kr_2)}$$

where subscripts indicate the amplitudes, and distances measured from the two point sources. Assuming the point sources are distance d apart, assigning $A = A_1 = -A_2$, and making the approximation that $r >> d$, one can approximate the pressure from two point sources of opposite phase as[2]:

$$p(r,t,\theta) = -j\frac{2A}{r}\sin\left(\tfrac{1}{2}kd\sin\theta\right)e^{j(\omega t - kr)}$$

where θ is an angle measured perpendicular to the line joining the two sources. This is the dipole approximation. This is just the equation for radiation from a point source multiplied by a factor dependent on θ and can be rewritten as:

$$p(r,t,\theta) = H(\theta)\frac{A}{r}e^{j(\omega t - kr)} \quad (1)$$

where:

$$H(\theta) = -2j\sin\left(\tfrac{1}{2}kd\sin\theta\right)$$

where $H(\theta)$ is called the *directionality* of the point source. Note that in order to express the radiation from a dipole in this form, the dipole far-field approximation $r >> d$ had to be made. Many different arrangements of point sources have had their radiation expressed in the form of Equation 1 (bipole, line source, line array, plane circular piston, etc) and can be found in the standard texts. For each of these a similar far-field approximation must be made in order to express the radiation in the form of Equation 1. The accepted general rule of thumb is that an acoustic source can be accurately represented in the form of Equation 1 when $r > S/\lambda$ where S is the radiating surface area, λ is the wavelength. The quantity S/λ is called the *Rayleigh Length*.

Again, since the acoustic wave equation is linear, the sound field radiated in the far-field by a collection of directional point sources is simply the summation of the radiation from each source:

$$p(r,t,\theta) = \sum_{k=1}^{N} H_k(\theta)\frac{A}{r_k}e^{j(\omega t - kr_k)}$$

where subscript k indicates the k-th directional point source. This is the *Directional Point Source* model.

The Directional Point Source model is the model used by Meyer Sound's MAPP Online, which is the program used to generate the Direction Point Source predictions in this paper. Though it is an approximation, and not valid in the near-field of each individual source, it has been found to be accurate and useful [3, 4, 5, 6].

Among the phenomena that the Directional Point Source model does not model is the "baffling" of one source by the physical presence of the others. This phenomena is modeled by the BEM model. The purpose of this paper is to compare the accuracy of the Directional Point Source and BEM models for practical arrays of loudspeakers.

1.2 Boundary Element Method

Summarizing the derivation of the Boundary Element Method is beyond the scope of this paper. The derivation can be found in a number of references [7, 8]. For the purposes of this paper it is sufficient to know that one can calculate the pressure at any point exterior or interior to a surface given one of three values at each point on that surface: the pressure, the particle velocity, or the impedance. For this paper the particle velocity was specified at each point on the surface of the loudspeaker and its woofers. The particle velocity was set to unity on the woofers, and to zero on the other surfaces of the loudspeaker.

Sysnoise is the program used to do the Boundary Element Method computations presented in this paper.

2 MEASUREMENTS

2.1 The Loudspeakers

Measurements and predictions were made for a small self powered line array speaker with two vented five inch cone drivers and three 0.75” metal dome tweeters coupled to a constant-directivity horn (Meyer Sound M1D). This loudspeaker has a frequency response which is flat between 75 Hz and 15 kHz to within ±4 dB. The two five inch drivers work in combination at low frequencies (60 Hz – 1000 Hz). At mid frequencies (1000 Hz – 1900 Hz) only one cone driver is fed from the crossover to maintain optimal polar and frequency response characteristics. Two of these loudspeakers are shown in Figure 1. They are 7.12 inches apart (183mm) when arrayed vertically.

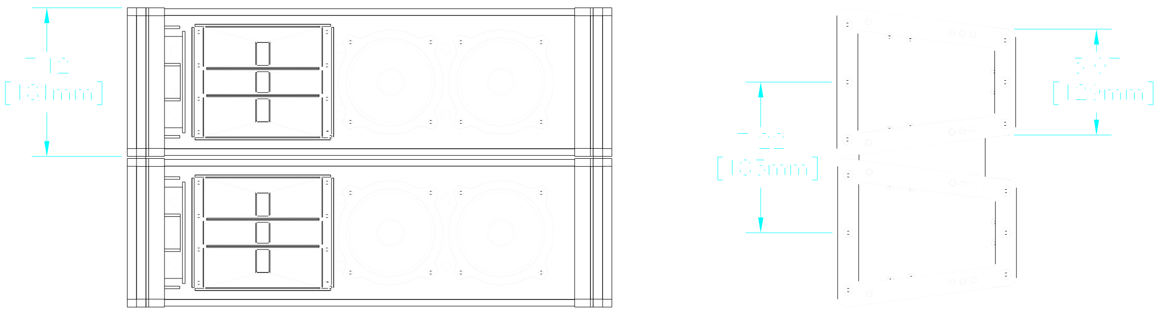

Figure 1 Two of the loudspeakers with five inch drivers used for measurement (Meyer Sound M1D) arrayed vertically.

Additional measurements were made with a larger self powered line array speaker with two vented ten inch cone drivers and one four inch high frequency compression driver (Meyer Sound M2D) shown in Figure 2. This loudspeaker has a frequency response which is flat between 70 Hz and 14 kHz to within ±4 dB. The two five inch drivers work in combination at low frequencies (60 Hz – 350 Hz). At mid frequencies (350 Hz – 575 Hz) only one cone driver is fed from the crossover to maintain optimal polar and frequency response characteristics. These loudspeakers are 12.37 inches apart (314mm) when arrayed vertically.

Figure 2 One of the loudspeakers with ten inch drivers (Meyer Sound M2D).

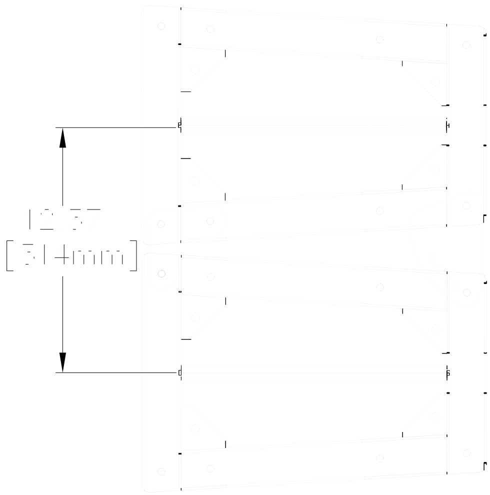

Figure 3 Two of the loudspeakers with ten inch drivers (Meyer Sound M2D) arrayed vertically.

2.2 Predictions of the Loudspeaker with five inch drivers

First a model was created of the two five inch drivers in their loudspeaker cabinet as shown in Figure 4. Next a model was created of those two drivers as they would be baffled if they were in the middle cabinet of a three cabinet array as shown in Figure 5.

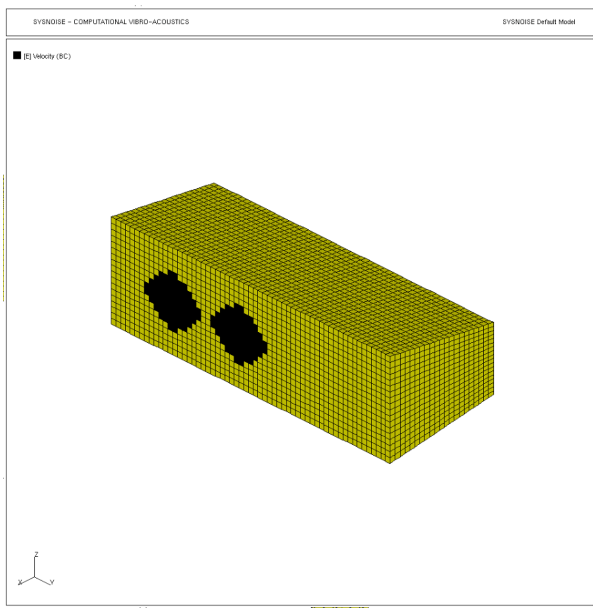

Figure 4 Sysnoise model of two five inch in their loudspeaker cabinet

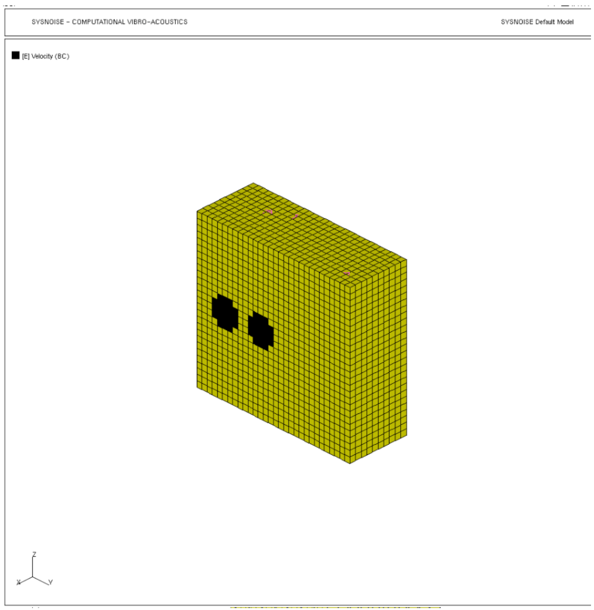

Figure 5 Two five inch drivers as they would be baffled by three cabinets

The on axis frequency response of the two woofers was predicted in Sysnoise for these two baffle conditions. Figure 6 shows the difference in dB of the on axis response between these two baffle conditions.

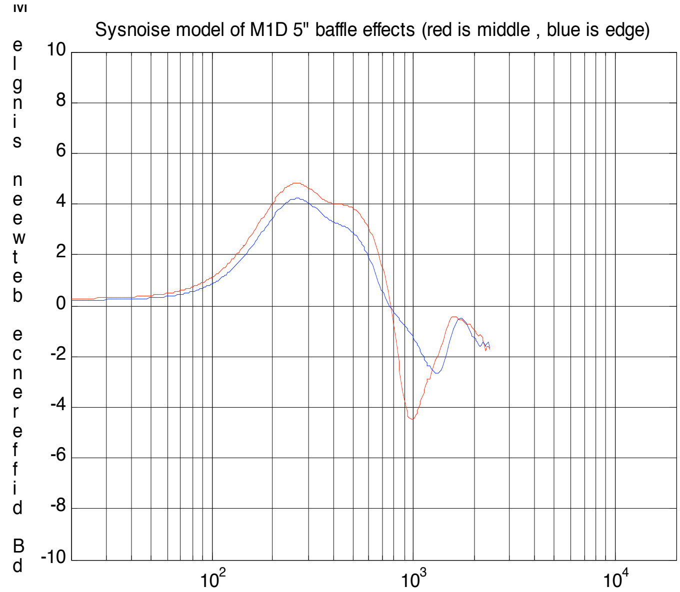

Figure 6 Difference in dB between the on axis baffle response of the two speakers under the two different baffle conditions

Next the polar response of the two five inch drivers was predicted in Sysnoise for these two baffle conditions as shown in Figure 7.

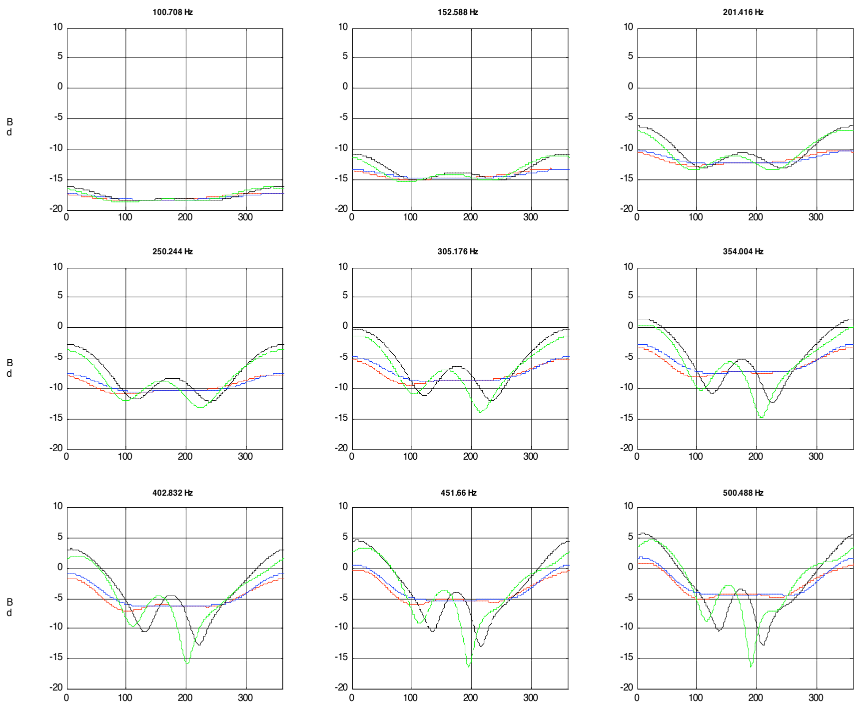

Figure 7 Polar Response for the two woofers as predicted by Sysnoise for the two baffle conditions

2.3 Comparing Predictions and Measurements of the loudspeaker with five inch drivers.

On axis frequency responses for arrays of the loudspeaker with five inch drivers were measured outdoors in a relatively anechoic area. First a single loudspeaker was placed level with the ground and measured on axis with a microphone placed on the ground. For comparison, a Directional Point Source prediction was made of two loudspeakers with zero degree splay. In other words, the reflection off the ground in the real measurements is being modelled in the Directional Point Source computations by an image source of the loudspeaker. Arrays of four and six ground stacked loudspeakers were measured and compared with Directional Point Source arrays of eight and twelve respectively. These data can be seen in Figure 8. Note that the ground at the location of these measurements is less than a perfect reflector above 1kHz, so that differences between the predicted and measured are not meaningful at the highest frequencies.

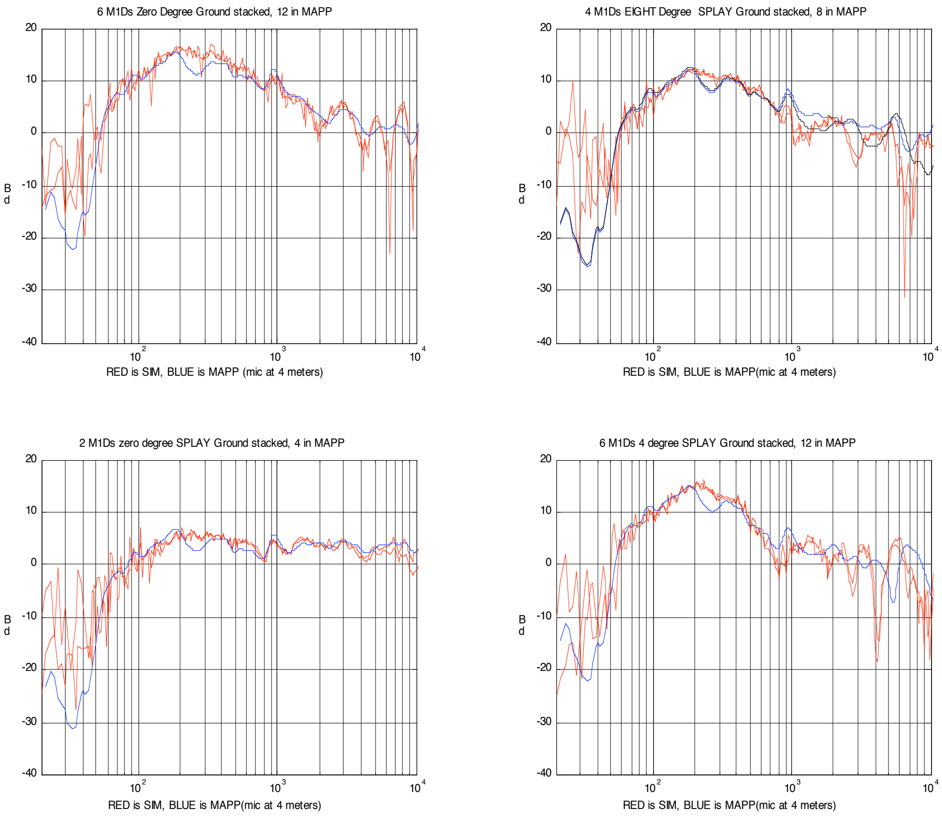

Figure 8 Comparison of Ground Stacked loudspeaker array to Directional Point Source model of loudspeaker array with twice the number of speakers.

2.4 Comparing Predictions and Measurements of the loudspeaker with ten inch drivers.

A concert hall (Zellerbach Hall) was rented and measurements and predictions were made of an array of six of the loudspeakers with ten inch drivers. Zellerbach Hall is a 2014-seat concert hall on the campus of the University of California at Berkeley. It has two balconies. As shown in Figure 9, multiple loudspeaker arrays were rigged for a variety of testing purposes. The array that was measured for this paper is the one in the upper right corner of Figure 9 containing six M2D loudspeakers and one M2D-Sub. The M2D-Sub was turned off in both the predictions and measurements. The microphone was placed approximately 40 feet away from the array.

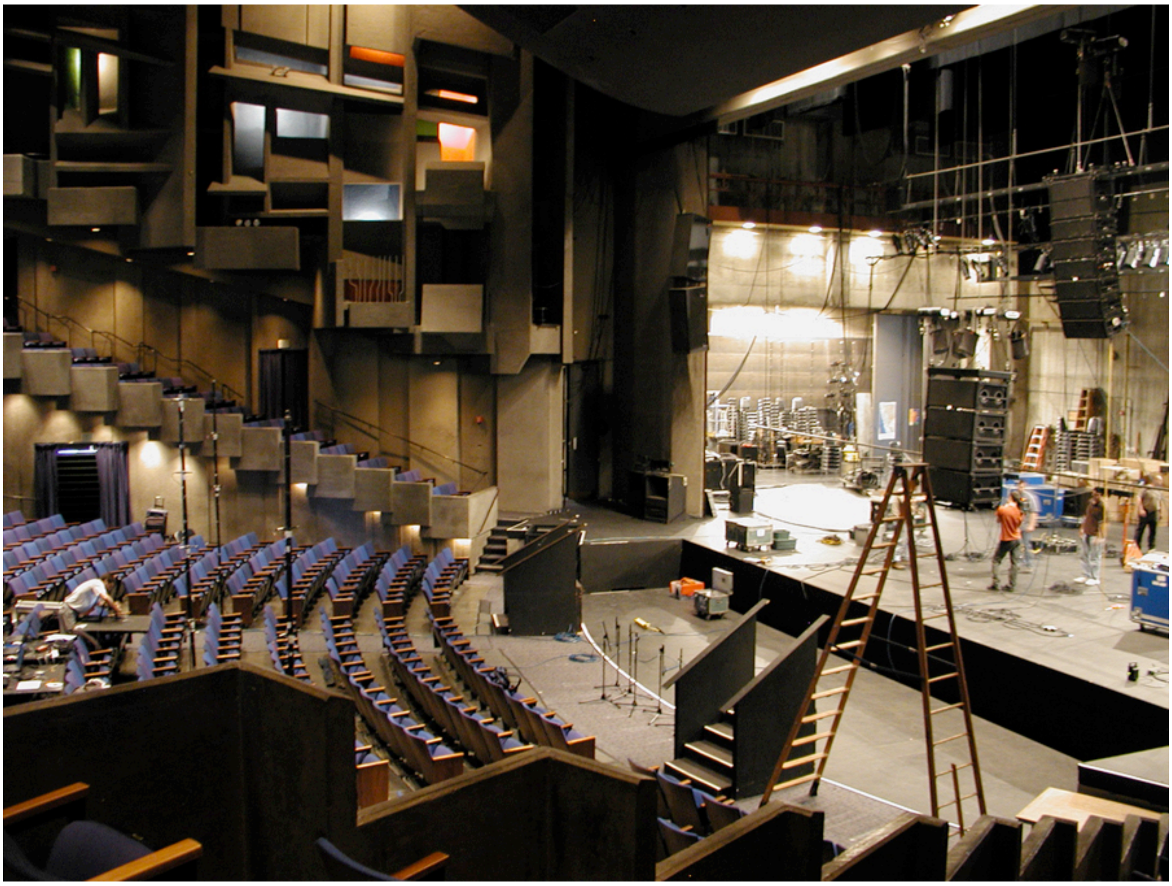

Figure 9 Zellerbach Hall with an array of six loudspeakers with ten inch drivers

Figure 10 shows the location of the loudspeakers and microphone with respect to the hall as they were predicted in MAPP Online. Also shown is the sound field in the 500Hz octave band.

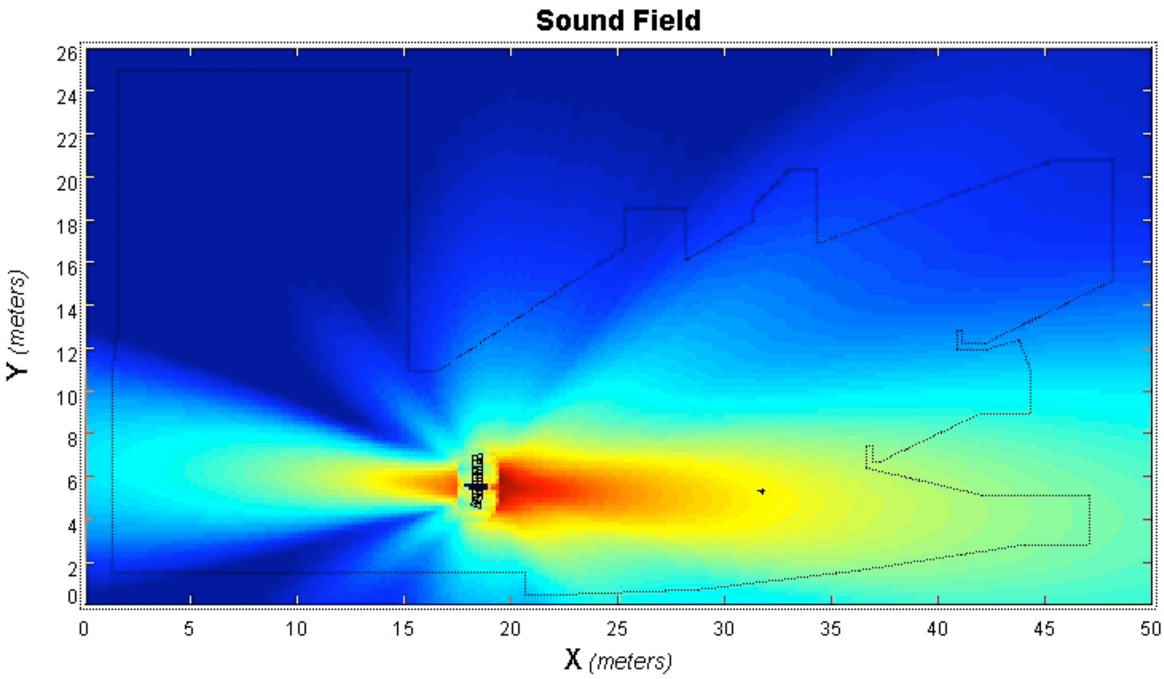

Figure 10 Location of loudspeaker array and microphone with respect to the hall, including 500Hz octave band sound field.

Figure 11 shows the frequency response as measured in the hall and predicted by MAPP Online.

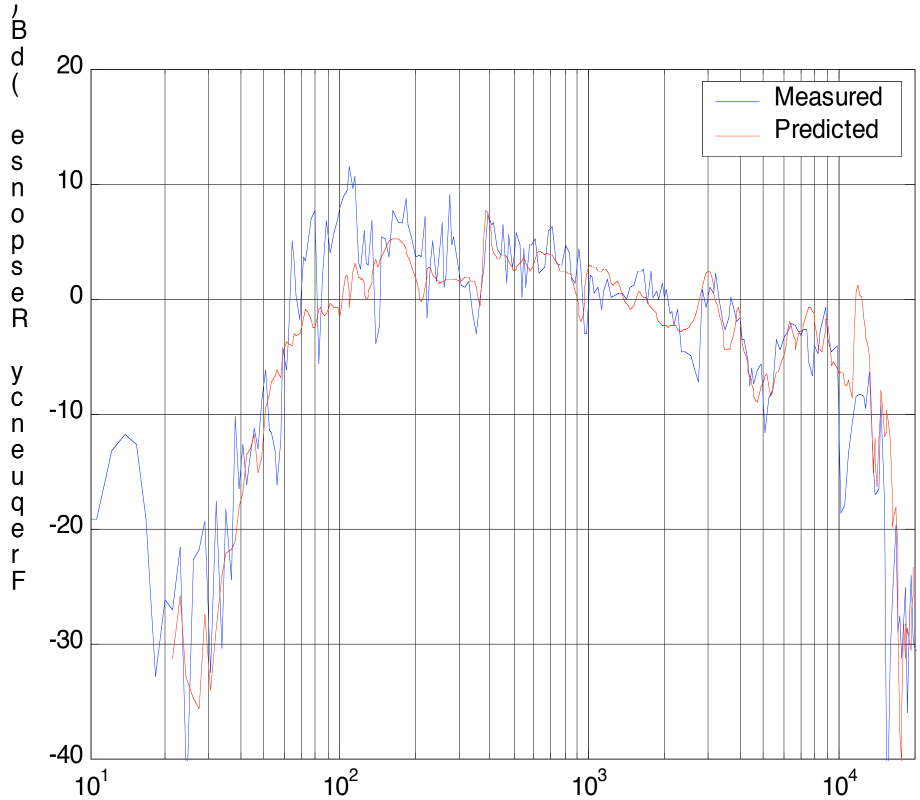

Figure 11 Frequency Response of array of six loudspeakers as measured in the hall and predicted by MAPP Online

The prediction and measurement match very well between 300Hz and 10kHz. Above 10kHz the measured frequency response was found to be sensitive to small changes in microphone position, and does not closely match the prediction. Below 300Hz room reverberation and mutual baffling of the loudspeakers increase the frequency response above what is predicted by the Directional Point Source model.

3 CONCLUSION

Individual loudspeakers, and arrays of that loudspeaker were predicted with the BEM method and the Directional Point Source model, and measured. BEM calculations for the loudspeaker with five inch drivers predicted an increase of 4dB centered around 300Hz due to baffling by other speakers. This same increase was measured with actual speakers.

An array of loudspeakers with ten inch drivers was predicted with the Directional Point Source model and measured in an actual concert hall. These measurements matched well between 300Hz and 10kHz. Below 300Hz room reverberation and mutual baffling of the loudspeakers increase the frequency response above what was predicted.

Though not perfect, the Directional Point Source model has been shown to be sufficiently accurate to be useful for loudspeaker system design, especially at frequencies above several hundred Hertz. Particularly advantageous is the fact that the Directional Point Source model can be computed in a matter of seconds for practical loudspeaker arrays, while the BEM calculations done for this paper each took several hours. The speed of the Directional Points Source model allows the user to design a sound system interactively, which is an enormous advantage over waiting over-night between each design iteration as is typical with BEM.

The mutual baffling of loudspeakers has been shown to make a measurable difference in the loudspeaker array frequency response. The BEM method has been shown to be able to predict this mutual baffling, and accurately match actual measurements.

Because mutual baffling has been shown to make a measurable difference, and because BEM has been shown to accurately predict it, In the future the authors will be investigating a hybrid methodology where the BEM method is used at low frequencies and the Direction Point Source method used at high frequencies.

4 REFERENCES

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